TE-Mode Propagation Properties of the Coupled Planar Kerr-Like Nonlinear Waveguides

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Abstract — **For the first time, a closed form expression of coupled Kerr-like nonlinear planar waveguides is derived for both guided and surface waves. The wave propagation properties along the Kerr-like nonlinear wavegudies are investigated analytically. The results show that the propagation properties depend on the permitted transmission powers and the initial excitation conditions. All the possible solutions can co-exist in the waveguide structures without mode-coupling.**

I. Introduction

Intensity dependent phenomena in nonlinear dielectric structures have attracted a great deal of interest in the past decade because nonlinear guided waves have potential, yet not fully explored, applications to optical signal processing for high-speed communications and computing [1 – 17]. While the intensity dependent nonlinear phenomena have been observed in semiconductor multiplayer systems, e.g. a sequence of GaAs-AlAs films exhibits very strong intensity dependent nonlinearity [16].

The guided nonlinear TE modes in a single slab Kerr-like nonlinear waveguide have been intensively investigated [1 – 16] numerically. Nonlinear surface waves on the interfaces of two nonlinear media have also been investigated [14,17]. However, to the author's knowledge, no any theoretical analysis in coupled kerr-like nonlinear planar waveguide has been systematically studied in the literature. In this paper, the TE-mode properties in coupled Kerr-like nonlinear planar waveguide structures are analytically investigated using the closed-form expression for the first time.

II. THEORY

The structure is shown in Fig.1, regions I, III and V are linear media and regions II and IV are Kerr-like nonlinear media with nonlinear coefficient α_2 and α_4 respectively. The propagation direction is z-direction with the form $e^{-j\beta z}$, where β is the propagation constant. The structure is uniform in y-direction, thus, $\frac{\partial}{\partial y} = 0$. For TE waves, $E_z \equiv 0$. From the Maxwell's equations it is known that now $E_x = 0$. Only E_y component of the electric field is non-zero. Let $E = E_y$.

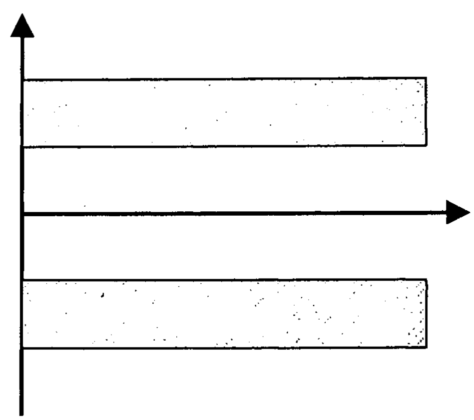

Fig. 1 The structure of a coupled Kerr-like nonlinear planar waveguide.

The wave equation is now [1-17]:

$$\frac{d^2 E}{dx^2} + k_0^2(\varepsilon_r - V)E = 0$$

with

$$V = (\frac{\beta}{k_0})^2, \qquad k_0^2 = \omega^2 \mu_0 \varepsilon_0$$

$$\varepsilon_r = \begin{cases} \varepsilon_1 & x \ge d \\ \varepsilon_2 + \alpha_2 E^2 & d \le x \le d(1+\delta_2) \\ \varepsilon_3 & |x| \le d \\ \varepsilon_4 + \alpha_4 E^2 & -d \ge x \ge -d(1+\delta_4) \\ \varepsilon_5 & x \le -d(1+\delta_4) \end{cases} \tag{1}$$

Regions I, III and V are linear and the solutions in these regions are well-known as:

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Region I:

$$E(x) = E_I \exp[-k_0 d\sqrt{V-\varepsilon_1}(\frac{x}{d(1+\delta_2)}-1)] \quad (2)$$

Region III:

$$E(x) = \begin{cases} A_3\cos(k_3x) + B_3\sin(k_3x) \\ A_3\cosh(\bar{k}_3x) + B_3\sinh(\bar{k}_3x) \end{cases} \quad (3a)$$

with

$$\begin{matrix} k_3^2 = k_0^2(\varepsilon_3 - V) & \varepsilon_3 > V \\ \bar{k}_3^2 = k_0^2(V-\varepsilon_3) & \varepsilon_3 < V \end{matrix} \quad (3b)$$

Region V:

$$E(x) = E_V \exp[k_0 d\sqrt{V-\varepsilon_5}(\frac{x}{d(1+\delta_4)}+1)] \quad (4)$$

Regions II and IV are nonlinear slabs, when $V>\varepsilon_2$ and meanwhile $V>\varepsilon_4$, the closed-form analytical solution can be found as:

Region II:

$$E(x) = \frac{A_2}{\cosh[\sqrt{\frac{\alpha_2}{2}}A_2(k_0d)\frac{x_2-x}{d}]} \quad (5)$$

and region IV:

$$E(x) = \frac{A_4}{\cosh[\sqrt{\frac{\alpha_4}{2}}A_4(k_0d)\frac{x_4-x}{d}]} \quad (6)$$

Where x_2 and x_4 are constants determined by the initial conditions. Parameter V must satisfy the condition simultaneously:

$$\frac{1}{2}\alpha_2A_2^2 + \varepsilon_2 = V = \frac{1}{2}\alpha_4A_4^2 + \varepsilon_4 \quad (7)$$

On the interfaces $x=d(1+\delta_2)$; $x=\pm d$; $x=-d(1+\delta_4)$, the electromagnetic field components E_y and H_z must be continuous. Using equations (2) – (7), the dispersion equation can be obtained. Here only the symmetrical case is presented because of the pager limitation.

III. RESULTS AND DISCUSSIONS

The symmetrical coupled Kerr-like nonlinear waveguides are studied in details because of the page constrains. The general solutions for other cases can be obtained from (1) – (7) straightforward.

Symmetrical coupled Kerr-like nonlinear guides:

$\varepsilon_2 = \varepsilon_4 = \varepsilon_r, \quad \alpha_2 = \alpha_4 = \alpha, \quad A_2 = A_4 = A$

$\varepsilon_1 = \varepsilon_5, \quad \delta_2 = \delta_4$

We can get

$x_2 = x_4 = 0$

$$k_0d = \frac{\tanh^{-1}\dfrac{\sqrt{V-\varepsilon_1}}{(1+\delta)\sqrt{\frac{\alpha}{2}}A}}{(1+\delta)\sqrt{\frac{\alpha}{2}}A} \quad (8)$$

$$V = (\frac{\beta}{k_0})^2 = \frac{\alpha}{2}A^2 + \varepsilon_r \quad (9)$$

It is obviously that the dispersion relationship depends on the nonlinearity, the electric field magnitude and the material. Paticularly, the propagation wavenumber determined by (8) and the dispersion determined by (9) are independent from the material parameter ε_3 of region III.

Fig. 2 indicates that for different thickness of the nonlinear slabs only the field magnitudes on the curves can exist in the waveguide structure.

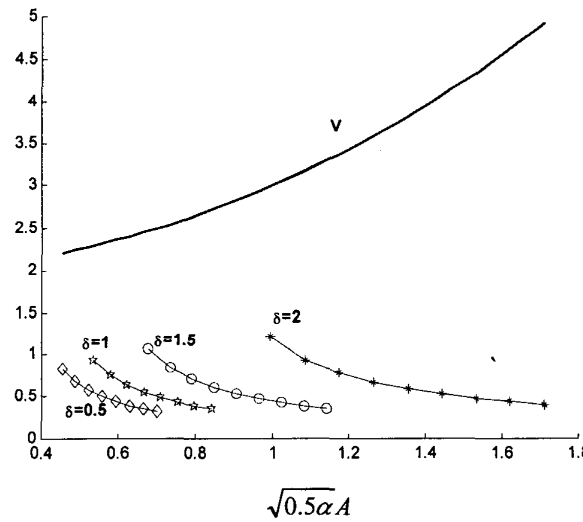

Fig. 2 Dispersion vs the field strength for different thickness of the nonlinear slabs. The solid-line is the curve V, and the symbols are the curves of $k_0d \sim \sqrt{0.5\alpha A}$.

Only those modes that satisfy (8) and (9) can propagate along the guides. It is not like the linear guides, where there are no any limitations on the field magnitudes. However, for a nonlinear coupled waveguide structure, the permitted modes depend not only on the material parameters, but also on the field strengths. For given materials parameters and the nonlinear coefficient, each given field magnitude has only one particular operating frequency and propagation constant β. If the field magnitude A changes, the propagation constant and the operating frequency will change also accordingly.

And the field distributions in the regions are:

Region I:

$$\frac{E(x)}{A}=\frac{\exp[-k_0 d\sqrt{V-\varepsilon_1}(\frac{x}{d(1+\delta)}-1)]}{\cosh[\sqrt{\frac{\alpha}{2}}A(1+\delta)(k_0 d)]} \tag{10a}$$

Region II:

$$\frac{E(x)}{A}=\frac{1}{\cosh[\sqrt{\frac{\alpha}{2}}A(k_0 d)\frac{x}{d}]} \tag{10b}$$

Region III:

$$\frac{E(x)}{A}=\frac{1}{\cosh[\sqrt{\frac{\alpha}{2}}A(1+\delta)(k_0 d)]}\times \begin{cases}\dfrac{\cos(k_0 d\sqrt{\varepsilon_3-V}\frac{x}{d})}{\cos(k_0 d\sqrt{\varepsilon_3-V})} & \varepsilon_3>V \\ \dfrac{\cosh(k_0 d\sqrt{V-\varepsilon_3}\frac{x}{d})}{\cosh(k_0 d\sqrt{V-\varepsilon_3})} & V>\varepsilon_3\end{cases} \tag{10c}$$

Region IV:

$$\frac{E(x)}{A}=\frac{1}{\cosh[\sqrt{\frac{\alpha}{2}}A(k_0 d)\frac{x}{d}]} \tag{10d}$$

Region V:

$$\frac{E(x)}{A}=\frac{\exp[k_0 d\sqrt{V-\varepsilon_1}(\frac{x}{d(1+\delta)}+1)]}{\cosh[\sqrt{\frac{\alpha}{2}}A(1+\delta)(k_0 d)]} \tag{10e}$$

For each given magnitude and the nonlinear coefficient, there is only one unique pair of k_0d and βd. That means that different initial excitation of the field will propagate in different velocity. All modes with different magnitudes and velocities can co-exist in the guided structure without cross-talk effects, because of the nonlinear effects. Suppose there are two modes existing in the structure, because only mode 1 and mode 2 can meet the wave equation (1) simultaneously, while mode 1 + mode 2 is not the solution of (1), mode 1 will not affect the behavior of the mode 2, vice versa. Therefore, nonlinear waveguide can be used to guide multi-mode propagations without mode-cross-coupling.

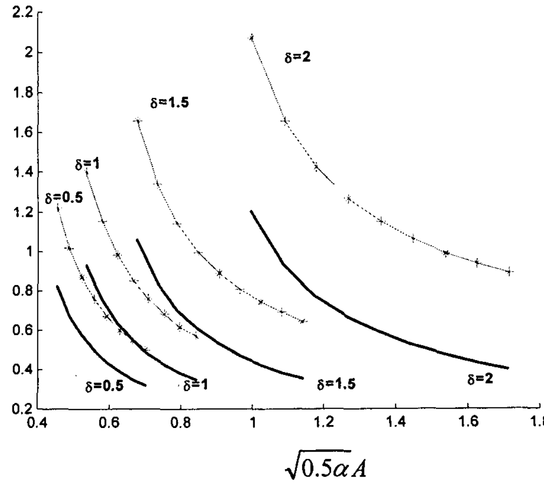

Fig. 3 The dispersion curves for different field magnitude and the thickness of the slabs. Solid-lines: k_0d; Symbols: βd.

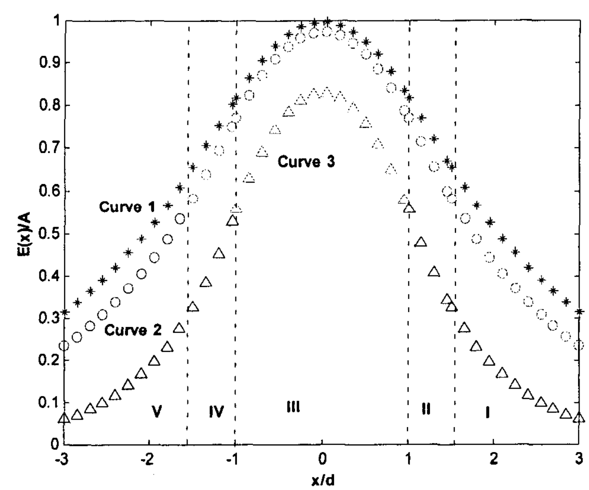

Fig. 4 The electric field distributions for different magnitudes, V, and k_0d. Curve 1: V=5.5678 and k_0d=0.3469; Curve 2: V=4.078 and k_0d=0.523; Curve 3: V=2.9889 and k_0d=1.1981;

The field distributions of the guided waves for different magnitudes, V and k_0d are depicted in

Fig. 4. For different propagation properties the field distributions are also different. While the surface waves are given in Fig. 5.

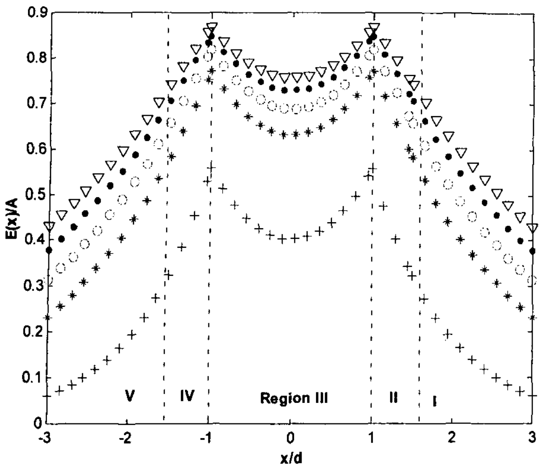

Fig. 5 The electric field distributions of the surface waves for different initial magnitudes, V, and k_0d. From top to bottom: V=83.8, 9.75, 5.57, 4.08, 1.58 and k_0d=0.06, 0.21, 0.35, 0.52, 0.49.

V. CONCLUSION

In this paper, TE waves in the structure consisting of TWO nonlinear Kerr-like slabs sandwiched in three linear media are studied analytically. Closed-form expressions for the dispersion relations as well the allowed wave-numbers of the TE waves are obtained for the first time. If the initial condition determines that the field maximum position is in the middle of the coupling structure, the substrate and clad linear media must have the same dielectric constant, and the two nonlinear media are identical. For given the value of the magnitude of the field strength only one mode with the particular operating frequency and the related propagation constant can be existed in the structure having the corresponding wave velocity. In addition, field distributions, propagation constant, maximum value of the field strength as well the operating frequency are dependent upon each other. The larger the magnitude of the wave is, the faster the wave will propagate. These results are useful for designing possible new devices based on the Kerr-like nonlinear waveguide structures.

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